EE3123 Tutorial 4 (Solution)
Power System Stability

Name: **Student No.:**

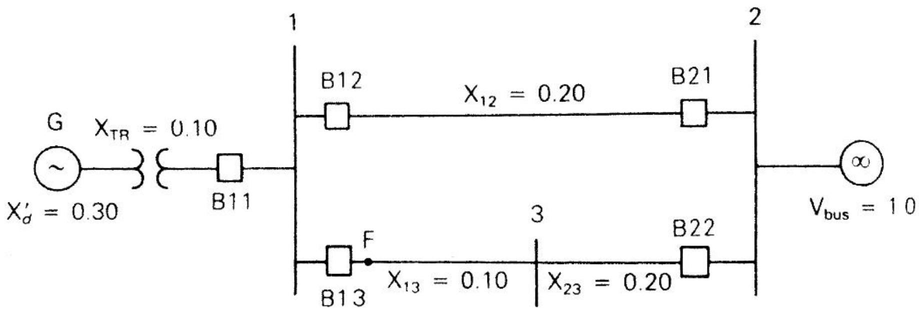
Q1

The synchronous generator below delivers 0.8 per-unit real power at 1.05 per-unit terminal voltage. Determine:

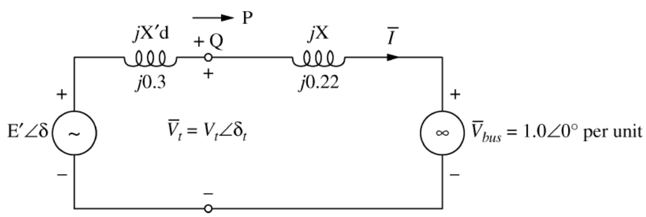
(a) the reactive power output of the generator;

(b) the generator internal voltage; and

(c) an equation for the electrical power delivered by the generator versus power angle δ.



Solution



(a) $P = \dfrac{V_t V_{bus}}{X}\sin\delta t \quad \Rightarrow \quad \sin\delta t = \dfrac{(P)(X)}{(V_t)(V_{bus})} = \dfrac{(0.8)(0.22)}{(1.05)(1.0)} = 0.167619$

$$\delta t = \sin^{-1}(0.167619) = 9.65°$$

$$\bar{I} = \frac{\bar{V}_t - \bar{V}_{bus}}{jX} = \frac{1.05\angle 9.65° - 1.0\angle 0°}{j0.22}$$

$$\bar{I} = \frac{0.03514 + j0.1760}{j0.22} = 0.81579\angle -11.291°$$

$$\bar{S} = \bar{V}_t I^* = (1.05\angle 9.65°)(0.81579\angle 11.291°) = 0.85658\angle 20.941°$$

$$\bar{S} = 0.80 + j0.306 \quad Q = \operatorname{Im}\bar{S} = \underline{\underline{0.306}} \text{ per unit}$$

(b) $\bar{E}' = \bar{V}_{bus} + j(X'_d + X)\bar{I} = 1.0\angle 0° + j(0.3 + 0.22)(0.81579\angle -11.291°)$

$$\bar{E}' = 1.0\angle 0° + 0.4242\angle 78.709°$$

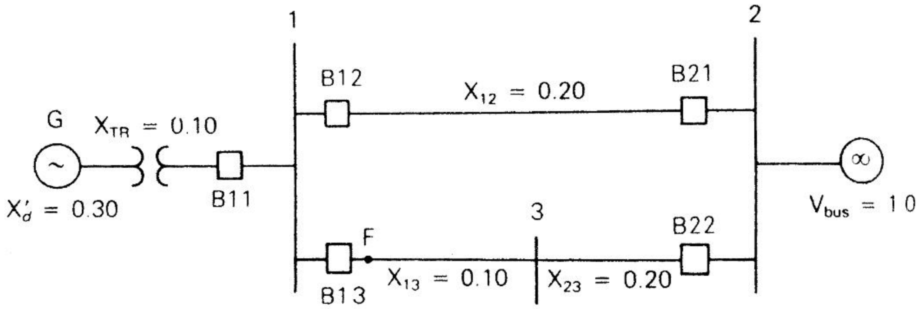
$$\bar{E}' = 1.160\angle 21.01° \text{per unit}$$

(c) $P = \frac{E' \, V_{bus}}{(X_d' + X)} \sin\delta = \frac{(1.160)(1.0)}{0.3 + 0.22} \sin\delta$

$P = 2.231 \sin\delta$ per unit

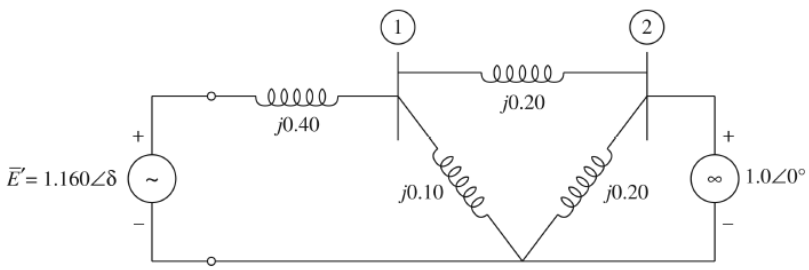
Q2

The generator in Q1 is initially operating in the steady-state condition given in Q1 when a three-phase-to-ground bolted short circuit occurs at bus 3. Determine an equation for the electrical power delivered by the generator versus power angle δ during the fault.



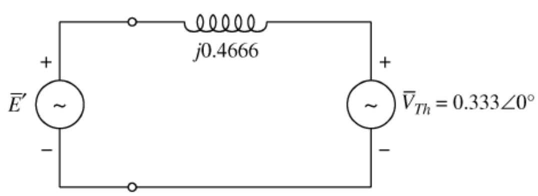
Solution

Circuit during the fault at bus 3:



where $\bar{E}' = 1.160\angle\delta$ is determined

The Thévenin equivalent, as viewed from the generator internal voltage source, shown here,

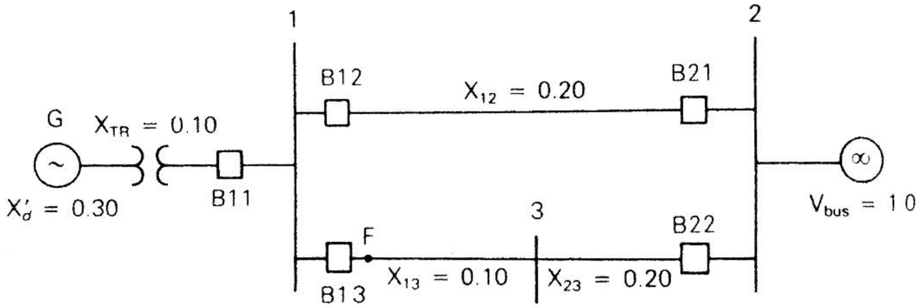


Thevenin Equivalent

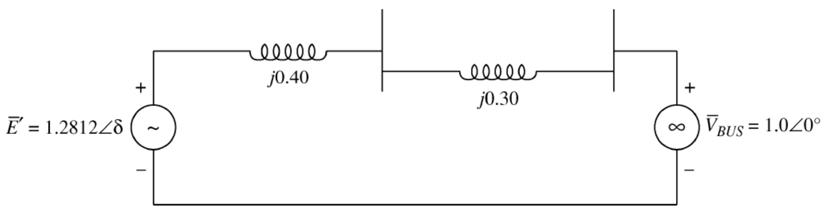
$$P = \frac{\bar{E}' \, V_{Th}}{x_{Th}} \sin\delta = \frac{(1.160)(0.333)}{0.4666} \sin\delta = \underline{\underline{0.8279}} \sin\delta \text{ per unit}$$

Q3

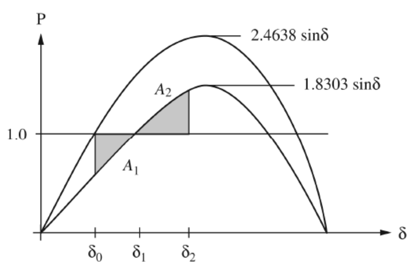
The 60-Hz synchronous generator shown below is initially operating in the steady-state condition: the infinite bus receives 1.0 per unit real power at 0.95 p.f. lagging. When circuit breaker B12 inadvertently opens, use the equal-area criterion to calculate the maximum value of the generator power angle δ. Assume $\omega_{p.u.}(t)=1.0$ in the swing equation.



Solution



$$P=\frac{E'V_{BUS}}{X_{eq}}\sin\delta=\frac{(1.2812)(1.0)}{0.70}\sin\delta=1.8303\sin\delta$$



$$\delta_0=\sin^{-1}\left(\frac{1}{2.4638}\right)=0.4179\,\text{rad}$$

$$\delta_1=\sin^{-1}\left(\frac{1}{1.8303}\right)=0.5780\,\text{rad}$$

$$A_1=\int_{\delta_0=0.4179}^{\delta_1=0.5780}(1.0-1.8303\sin\delta)\,d\delta=\int_{\delta_1=0.5780}^{\delta_2}(1.8303\sin\delta-1)\,d\delta=A_2$$

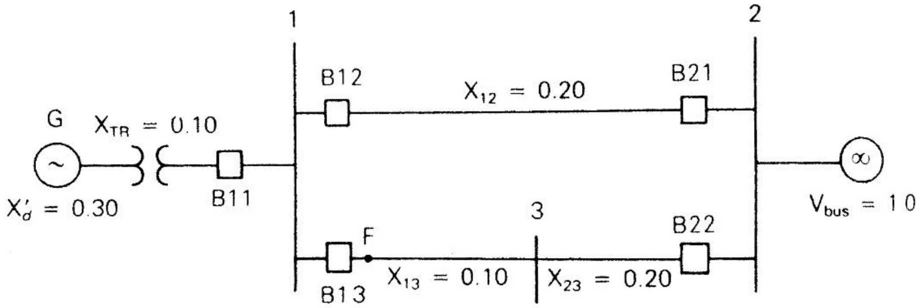
$$(0.5780-0.4179)+1.8303(\cos 0.5780-\cos 0.4179)=1.8303(\cos .5780-\cos\delta_2)$$
$$-(\delta_2-0.5780)$$

$$1.8303\cos\delta_2+\delta_2=2.0907$$

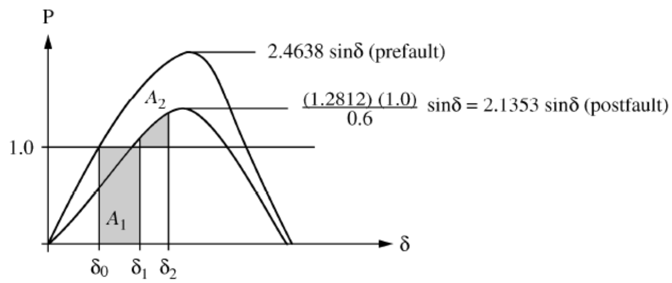
Solving iteratively (Newton-Raphson) $\delta_2=\underline{\underline{0.7439}}\,\text{rad}=\underline{\underline{42.62}}°$

Q4

The 60-Hz synchronous generator shown below is initially operating in the steady-state condition: the infinite bus receives 1.0 per unit real power at 0.95 p.f. lagging. There is a temporary three-phase-to-ground short circuit that occurs at point F. Exact three cycles later, circuit breakers B13 and B22 permanently open to clear the fault. Use the equal-area criterion to determine the maximum value of the power angle δ_2. (H = 3 pu-s)



Solution



$$\delta_0 = 0.4179\,\text{rad}$$
$$\delta_1 = 0.4964\,\text{rad}$$

$$A_1 = \int_{\delta_0=0.4179}^{\delta_1=0.4964} 1.0\,d\delta = \int_{\delta_1=0.4964}^{\delta_2} (2.1353\sin\delta - 1.0)\,d\delta$$

$$(0.4964 - 0.4179) = 2.1353(\cos 0.4964 - \cos\,\delta_2) - (\delta_2 - 0.4964)$$

$$2.1353\cos\delta_2 + \delta_2 = 2.2955$$

Solving iteratively using Newton Raphson with $\delta_2(0) = 0.60\,\text{rad}$

$$\delta_2(i+1) = \delta_2(i) + [-2.1353\sin\delta_2(i) + 1]^{-1}[2.2955 - 2.1353\cos\delta_2(i) - \delta_2(i)]$$

i	0	1	2	3	4
δ_2	0.60	0.925	0.804	0.785	0.7850

$$\delta_2 = 0.7850\,\text{rad} = 44.98°$$